Advanced Econometrics II
Homework 4

Deadline: 2012-02-07 09:30

You are to hand in your homework via email to ta@zamojski.net by 9:30 on Tuesday. I would appreciate if your answers were TeXed in full, if you insist you can scan your handwritten answers at TI and send them to me as well.

You can work in groups of two.

If there is a computational exercise included in the homework, the quality of your coding will be judged (commenting, efficiency, etc.). Your code should be easy to read. Things that make it easier: lots of comments (e.g. explaining what loops are meant to do), camel case variable names (e.g. mErrorsUniformlyDistributed), consistency. You are to include your code in the body of the report, e.g. if you are TeXing your answers then with the listings package. You can use Ox, Python, C++, or Matlab. If I am not able to run your code after extracting your answers (assuming they are zipped) to a separate folder you will lose points. Looking ahead, it is in your best interest to combine Ox (computations, sometimes graphics) and Python (database management, multiprocessing management, graphics) as it will cut the time needed for simulations considerably. In the empirical exercises you are expected to provide comments for your results and methods (e.g tests) used.

Exercise 1

Consider the linear regression model with serially correlated errors,

$$y_t = \beta_0 + \beta_1 x_t + u_t \tag{1}$$

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{2}$$

where the ε_t are IID, and the autoregressive parameter ρ is assumed either to be known or to be estimated consistently. The explanatory variable x_t is assumed to be contemporaneously correlated with ε_t. The covariance matrix of u is $\mathbf{\Omega}$. Express this model in the form (9.20) without taking account of the first observation.

Let Ω_t be the information set for observation t with $\mathrm{E}[\varepsilon_t \mid \Omega_t] = 0$. Suppose that there exists a matrix $\mathbf{Z}$ of instrumental variables, with $Z_t \in \Omega t$, such that the explanatory vector $\mathbf{x}$ with typical element x_t is related to the instruments by the equation

$$\mathbf{x} = \mathbf{Z}\pi + \mathbf{v} \tag{3}$$

where $\mathrm{E}[v_t \mid \Omega_t] = 0$. Derive the explicit form of the expression $\left(\mathbf{\Psi}^{\mathrm{T}}\mathbf{X}\right)_t$ defined implicitly by equation (9.24) for this model. Find a matrix $\mathbf{W}$ of instruments that satisfy the predeterminedness condition in the form (9.30) and that lead to asymptotically efficient estimates of the parameters β_0 and β_1 computed on the basis of the theoretical moment conditions (9.31) with your choice of $\mathbf{W}$.

Exercise 2

The minimization of the GMM criterion function (9.87) yields the estimating equations (9.89) with $\mathbf{A} = \mathbf{A}^{\mathrm{T}}\mathbf{W}$. Assuming that the $n \times l$ instrument matrix $\mathbf{W}$ satisfies the predeterminedness condition in the form (9.30), show that these estimating equations are asymptotically equivalent to the equations:

$$\bar{\mathbf{F}}_{\mathbf{0}}^{\mathrm{T}}\mathbf{\Psi}\mathbf{P}_{\mathbf{\Psi}^{\mathrm{T}}\mathbf{W}}\mathbf{\Psi}^{\mathrm{T}}\mathbf{f}\left(\hat{\theta}\right) \tag{4}$$

where, as usual, $\bar{\mathbf{F}} \equiv \bar{\mathbf{F}}\left(\theta_{\mathbf{0}}\right)$, with $\theta_{\mathbf{0}}$ the true parameter vector. Next, derive the asymptotic covariance matrix of the estimator defined by these equations. Show that these equations are the optimal estimating equations for an overidentified estimation based on the transformed zero functions $\mathbf{\Psi}^{\mathrm{T}}\mathbf{f}\left(\theta\right)$ and the transformed instruments $\mathbf{\Psi}^{\mathrm{T}}\mathbf{W}$. Show further that, if the condition $S\left(\mathbf{F}_{\mathbf{0}}\right) \subseteq S\left(\mathbf{W}\right)$ is satisfied, the asymptotic covariance matrix of the estimator obtained by solving these equations coincides with the optimal asymptotic covariance matrix (9.83).

Exercise 3

Map the OLS regressions into the GMM framework (in each case state: moment conditions, parameter estimator, variance estimator, and variance of the parameter estimator). Consider three cases: serially uncorrelated and homoskedastic errors vs. heteroskedastic errors vs. serially correlated errors.