Advanced Econometrics II
Homework 5

Deadline: 2012-02-14 09:30

You are to hand in your homework via email to ta@zamojski.net by 9:30 on Tuesday. I would appreciate if your answers were TeXed in full, if you insist you can scan your handwritten answers at TI and send them to me as well.

You can work in groups of two.

If there is a computational exercise included in the homework, the quality of your coding will be judged (commenting, efficiency, etc.). Your code should be easy to read. Things that make it easier: lots of comments (e.g. explaining what loops are meant to do), camel case variable names (e.g. mErrorsUniformlyDistributed), consistency. You are to include your code in the body of the report, e.g. if you are TeXing your answers then with the listings package. You can use Ox, Python, C++, or Matlab. If I am not able to run your code after extracting your answers (assuming they are zipped) to a separate folder you will lose points. Looking ahead, it is in your best interest to combine Ox (computations, sometimes graphics) and Python (database management, multiprocessing management, graphics) as it will cut the time needed for simulations considerably. In the empirical exercises you are expected to provide comments for your results and methods (e.g tests) used.

Exercise 1

State any necessary assumptions, comment on what happens when they are not met, and prove:

1. Score equality
$$\mathrm{E}\left[s\left(\theta_0, y\right)\right] = 0 \tag{1}$$
2.
$$\mathrm{E}\left[l\left(\theta_0, y\right)\right] - \mathrm{E}\left[l\left(\theta, y\right)\right] > 0 \tag{2}$$
3. Information (matrix) equality
$$\mathcal{J}\left(\theta\right) = -\mathcal{H}\left(\theta\right) \tag{3}$$

Exercise 2 Cramer-Rao inequality

1. Let $\bar{\theta}$ denote any unbiased estimator of the k parameters of a parametric model fully specified by the loglikelihood function $l\left(\theta\right)$. The unbiasedness property can be expressed as the following identity
$$\int L\left(y, \theta\right) \bar{\theta} dy = \theta \tag{4}$$

By using the relationship between $L(\theta, y)$ and $l(\theta, y)$ and differentiating this identity with respect to the components of θ, show that

$$\underbrace{cov_\theta\left(g(\theta), (\bar{\theta} - \theta)\right)}_{\text{Calculated under the DGP characterized by } \theta} = I_k \tag{5}$$

Let V denote the $2k \times 2k$ covariance matrix of the $2k$-vector obtained by stacking the components of $g(\theta)$ above the components of $\bar{\theta} - \theta$. Partition this matrix into $4k \times k$ blocks as follows

$$V = \begin{bmatrix} V_1 & C \\ C^T & V_2 \end{bmatrix} \tag{6}$$

Where V_i are the covariance matrices of the vectors $g(\theta)$ and $\bar{\theta} - \theta$ under the DGP characterized by θ. Use the fact that V is positive semidefinite to show that the difference between V_2 and $I^{-1}(\theta)$, where $I(\theta)$ is the information matrix for the model, is a positive semidefinite matrix. You can use results of Exercise 7.11 without deriving them.

2. Apply this reasoning to show a single parameter proof of the lower bound.

Exercise 3

For a sample of n observations y_t generated from the exponential distribution, the loglikelihood function is (10.04), and the MLE is (10.06). Derive the asymptotic information matrix $\mathcal{J}(\theta)$ (a scalar in this case). Use this to derive the asymptotic distribution of of $\sqrt{n}\left(\hat{\theta} - \theta_0\right)$. Derive the empirical Hessian estimator of the variance of $\hat{\theta}$. What is the IM estimator?

There is an alternative parametrization of the exponential distribution, in which the parameter is $\phi \equiv \frac{1}{\theta}$. Write down the logliklihood function in terms of ϕ and obtain the asymptotic distribution of $\sqrt{n}\left(\hat{\phi} - \phi_0\right)$. What is the empirical Hessian estimator of the variance of $\hat{\phi}$? What is the IM estimator?